Kernel on Bag of Paths For Measuring Shapes Similarity

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Introduction

Context :

- shape recognition,
- structured representation,
- application to shock graph retrieval.

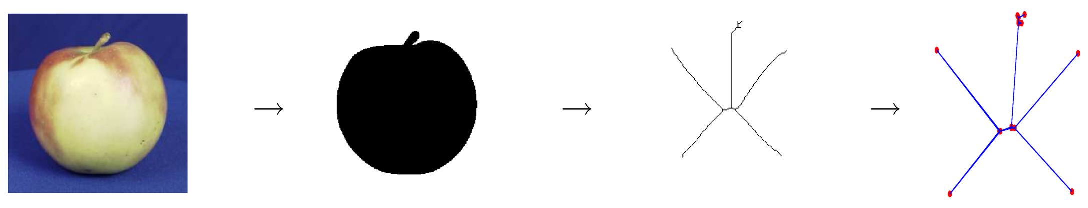
Using graph for comparison :

- shape represented with skeleton,
- skeleton comparison ⇔ graph comparison,
- graph kernel : kernel on set approach,
- graph : set of subgraphs, set of paths.

Shape representation with graph

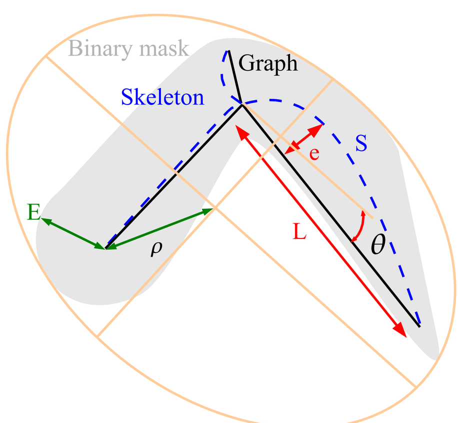
How to build a graph ?

1. original image,
2. binary mask,
3. morphological skeleton,
4. transform skeleton into graph :
   - vertices : skeleton branch ending or branches intersection,
   - edges.



5. label graph :
   edge :
   - length (L),
   - skeleton length (s),
   - orientation (θ),
   - distance between skeleton and edge (e).

   vertex :
   - distance to the center of mass (ρ),
   - distance to the nearest shape edge (E).



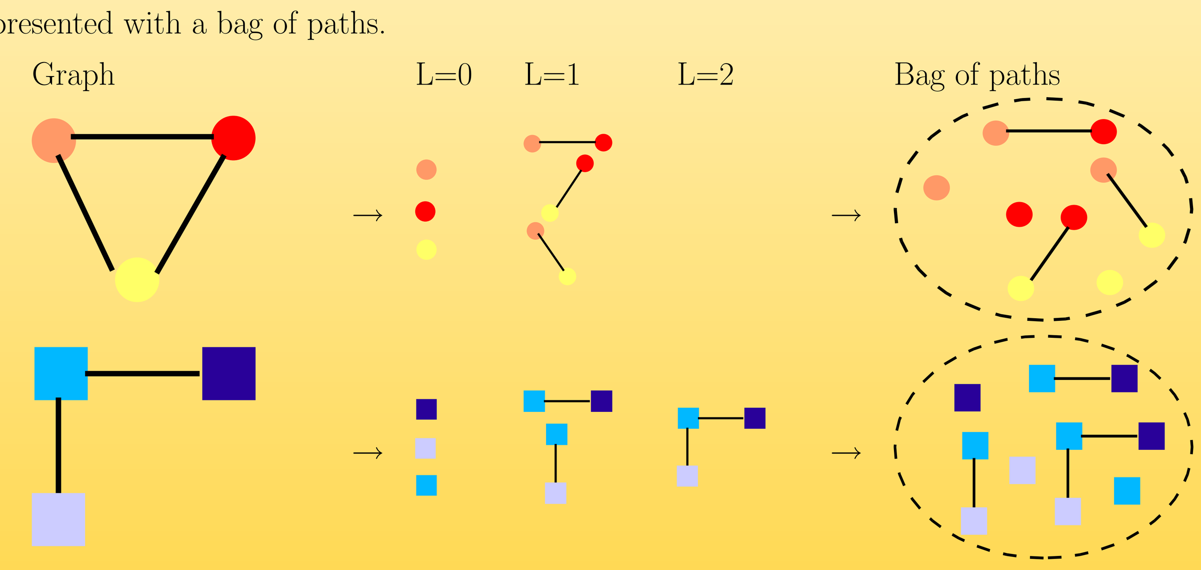
Set of paths

- V : set of vertices,
- E : set of edges,
- labeled graph : $G = (V, E)$,
- path : $h = \{v_1, \cdots, v_n\}$
  - generated using random walk or shortest path,
  - threshold for path length.

⇒ h is a subgraph of G

- set of m paths : $P = \{h_1, h_2, ..., h_m\}$

⇒ G is represented with a bag of paths.



Similarity between path :

$$K_L(h, h') = \begin{cases} 0 & \text{, if path lengths are different} \\ K_v(l(v_1), l(v_1')) \prod_{i=2}^{n} K_e(l(v_{i-1}, v_i), l(v_{i-1}', v_i')) K_v(l(v_i), l(v_i')) & \text{, otherwise} \end{cases}$$

- K_v kernel on vertex label,
- K_e kernel on edge label.

References

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Graph kernel

Wallraven [6] : building a kernel on set ⇔ computing a similarity between each element of sets :

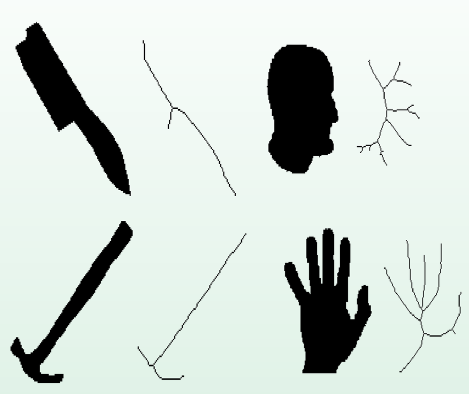
$$K(G, G') = \sum_{h} \sum_{h'} K_L(h, h')$$

- **Kashima [4]:** $K(G_1, G_2) = \sum_{h_1, h_2 \in V_1^\star, V_2^\star} p_1(h_1) p_2(h_2) K_L(h_1, h_2)$
  - p_1, p_2 : probabilities to generate random walk,
  - positive definite.
- **Mean kernel :** $K(G_1, G_2) = K(P_1, P_2) = \frac{1}{N_1}\frac{1}{N_2} \sum_{i:h_i \in P_1} \sum_{j:h_j \in P_2} K_L(h_i, h_j)$
  - all paths are compared,
  - positive definite.
- **Max matching kernel :** $K(G_1, G_2) = K(P_1, P_2) = \frac{1}{2}[\hat{K}(P_1, P_2) + \hat{K}(P_2, P_1)]$
  - best comparison retained,
  - $\hat{K}(P_1, P_2) = \frac{1}{|P_1|} \sum_{i:h_i \in P_1} \max_{j:h_j \in P_2} K_L(h_i, h_j)$
- **Matching kernel :** $\hat{K}(P_1, P_2) = \frac{1}{|P_1|}\frac{1}{|P_2|} \sum_{i:h_i \in P_1} \sum_{j:h_j \in P_2} K_{d_L}(h_i, h_j)$
  - Haasdonk [3] : $\max_{j:h_j \in P_2} K_L(h_i, h_j)$ approximated with $\sum_{j:h_j \in P_2} K_{d_L}(h_i, h_j)$
  - $K_{d_L}(h_1, h_2) = \exp\left(-\frac{d_L(h_1,h_2)^2}{2\sigma^2}\right)$,
  - $d_L(h_1, h_2)^2 = K_L(h_1, h_1) + K_L(h_2, h_2) - 2K_L(h_1, h_2)$.
  - positive definite for all $\sigma > 0$.
- **Path level set kernel :** $K(G_1, G_2) = K(P_1, P_2) = \langle b_{P_1}, b_{P_2} \rangle \cdot \sum_{i:h_i \in P_1} \sum_{j:h_j \in P_2} \alpha_i^{P_1} \alpha_j^{P_2} K_L(h_i, h_j)$
  - Desobry [2] : comparing the probability distribution of each set of paths using one-class SVM,
  - optimization problem : $\min_{f \in \mathcal{H}, b} \frac{1}{2}\|f\|_{\mathcal{H}}^2 + \frac{1}{\nu n} \sum_i \max(0, b - f(x_i)) - b$
  - $f_{P_1}(h) = \sum_i \alpha_i^{P_1} K_L(h_i, h) - b_{P_1}$ and $f_{P_2}(h) = \sum_j \alpha_j^{P_2} K_L(h_j, h) - b_{P_2}$
  - positive definite.

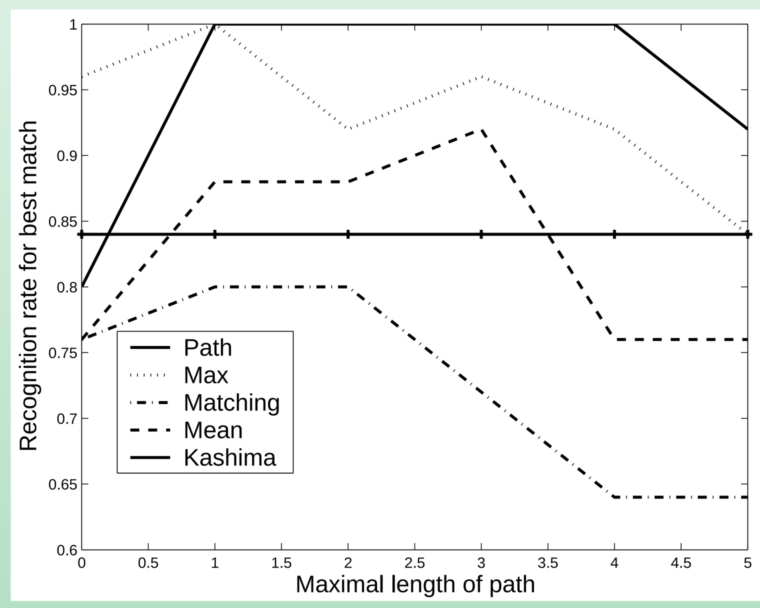
Shock graph mining

- Rudger tools database :25 objects, 8 classes,
- shape retrieval : ranking for each shape query,
- vertex label : E, ρ,
- edge label : L.

→ compare graph formulations
→ path length influence



Performance of each graph kernel according to path length :



- Sebastian [5], Demirci [1] : 96%,
- path-level set, max matching kernel : 100%,
- path length : 2 or 3 for best performance.

Query result for each object of the database :

Conclusion

Conclusions :

- Graph kernel for measuring graph similarity,
- New graph kernel formulations,
- Application to shock graph mining.

Perspectives :

- Add other information : local histograms, texture,
- Explore other ways for graph designing : points of interest.